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LINEAR PRIME LABELING OF SOME DIRECT TREE GRAPHS

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ABSTRACT

Linear prime labeling of a graph is the labeling of the vertices with {0,1,2---,p-1} and the direct edges with twice the value of the terminal vertex plus value of the initial vertex. The greatest common incidence number of a vertex (gcin) of in degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the gcin of each vertex of in degree greater than one is one, then the graph admits linear prime labeling. Here we investigate some direct tree graphs for linear prime labeling.

Keywords*: Graph labeling, linear, prime labeling, prime graphs, direct graphs, tree.*

I. INTRODUCTION

All graphs in this paper are finite di graphs. The direction of the edge is from v_i to v_j iff $f(v_i) < f(v_j)$. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [1],[2],[3] and [4] . Some basic concepts are taken from Frank Harary [2]. In this paper we investigated the linear prime labeling of some direct tree graphs.

Definition: 1.1 Let G be a graph with p vertices and q edges. The greatest common incidence number (***gcin***) of a vertex of in degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

Definition 1.2 A Graph is said to be a di graph if each edge of G has a direction**.**

Definition 1.3 In-degree of a vertex in a digraph is the number of edges incident at that vertex.

II. MAIN RESULTS

Definition 2.1 Let G = (V(G),E(G)) be a di graph with p vertices and q edges . Define a bijection
f : V(G) → {0,1,2,---, p-1} by $f(v_i)$ = i-1, for every i from 1 to p and define a 1-1 mapping
f_{lpl}^{*} : E(G) → set of natural numbers N by $f_{lpl}^{*}(v_i v_j) = f(v_i) + 2f(v_j)$ for every direct edge v_iv_j. The induced function f_{lpl}^{*} is said to admit linear prime labeling, if for each vertex of in degree at least 2, the ***gcin*** of the labels of the incident edges is 1.

Definition 2.2 A graph which admits linear prime labeling is called linear prime graph.

Definition 2.3 Let G be the graph obtained by joining pendant edges alternately to the vertices of a path P_n (n > 3). G is denoted by P_n Θ A(K_1).

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Theorem 2.1 Direct comb graph $P_n \odot K_1$ (n > 2) admits linear prime labeling.

Proof: Let G = $P_n \odot K_1$ and let $v_1, v_2, ---, v_{2n}$ are the vertices of G.
Here |V(G)| = 2n and |E(G)| = 2n-1.
Define a function f : V → {0,1,2,---,2n-1} by

$$f(v_i) = i-1, \; i = 1,2,---,2n$$

Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f_{lpl}^* is defined as follows

$f_{lpl}^*(v_{2i-1}\, v_{2i}) = 6i-4, \qquad i = 1,2,---,n.$
$f_{lpl}^*(v_{2i}\, v_{2i+2}) = 6i+1, \qquad i = 1,2,---,n-1.$

Clearly f_{lpl}^* is an injection.

gcin of (v_{2i+2}) $= \gcd \text{ of } \{f_{lpl}^*(v_{2i}\, v_{2i+2}),\ f_{lpl}^*(v_{2i+1}\, v_{2i+2})\}$
$= \gcd \text{ of } \{6i+1, 6i+2\}$
$= 1, \qquad i = 1,2,---,n-1.$

So, ***gcin*** of each vertex of in degree greater than one is 1.
Hence $P_n \odot K_1$, admits linear prime labeling.

Example 2.1 G = $P_n \odot K_1$.

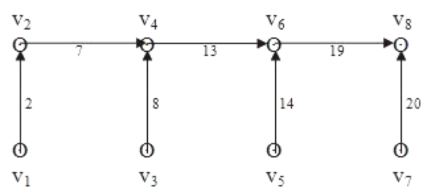

fig – 2.1

Theorem 2.2 Direct centipede graph $P_n \odot 2K_1$ (n > 2) admits linear prime labeling.

Proof: Let G = $P_n \odot 2K_1$ and let $v_1, v_2, ---, v_{3n}$ are the vertices of G.
Here |V(G)| = 3n and |E(G)| = 3n-1.
Define a function f : V → {0,1,2,---,3n-1} by

$$f(v_i) = i-1, \; i = 1,2,---,3n$$

Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f_{lpl}^* is defined as follows

$f_{lpl}^*(v_{3i-2}\, v_{3i-1}) = 9i-7, \qquad i = 1,2,---,n.$
$f_{lpl}^*(v_{3i-2}\, v_{3i}) = 9i-5, \qquad i = 1,2,---,n.$
$f_{lpl}^*(v_{3i-2}\, v_{3i+1}) = 9i-3, \qquad i = 1,2,---,n-1.$

Clearly f_{lpl}^* is an injection.
In degree of each vertex is less than 2.
Hence $P_n \odot 2K_1$, admits linear prime labeling.

Example 2.2 G = $P_4 \odot 2K_1$.

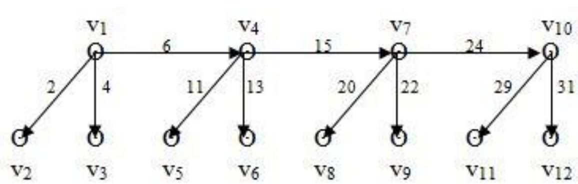

fig – 2.2

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Theorem 2.3 Direct hurdle graph Hd_n ($n > 3$) admits linear prime labeling.

Proof: Let $G = Hd_n$ and let $v_1, v_2, ---, v_{2n-2}$ are the vertices of G.
Here $|V(G)| = 2n-2$ and $|E(G)| = 2n-3$.
Define a function $f : V \to \{0,1,2,---,2n-3\}$ by

$$f(v_i) = i-1,\ i = 1,2,---,2n-2$$

Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f_{lpl}^* is defined as follows

$f_{lpl}^*(v_1\, v_2) = 2.$
$f_{lpl}^*(v_{2i}\, v_{2i+1}) = 6i-1,$ $i = 1,2,---,n-2.$
$f_{lpl}^*(v_{2i}\, v_{2i+2}) = 6i+1,$ $i = 1,2,---,n-2.$

Clearly f_{lpl}^* is an injection.
In degree of each vertex is less than 2.
Hence Hd_n, admits linear prime labeling.

Example 2.3 $G = Hd_4$

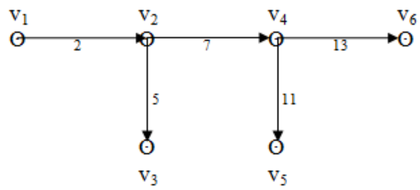

fig – 2.3

Theorem 2.4 Direct twig graph $T_w(n)$ ($n > 3$) admits linear prime labeling.

Proof: Let $G = T_w(n)$ and let $v_1, v_2, ---, v_{3n-4}$ are the vertices of G.
Here $|V(G)| = 3n-4$ and $|E(G)| = 3n-5$.
Define a function $f : V \to \{0,1,2,---,3n-5\}$ by

$$f(v_i) = i-1,\ i = 1,2,---,3n-4$$

Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f_{lpl}^* is defined as follows

$f_{lpl}^*(v_1\, v_2) = 2.$
$f_{lpl}^*(v_{3i-1}\, v_{3i}) = 9i-4,$ $i = 1,2,---,n-2.$
$f_{lpl}^*(v_{3i-1}\, v_{3i+1}) = 9i-2,$ $i = 1,2,---,n-2.$
$f_{lpl}^*(v_{3i-1}\, v_{3i+2}) = 9i,$ $i = 1,2,---,n-2.$

Clearly f_{lpl}^* is an injection.
In degree of each vertex is less than 2.
Hence $P_n \odot 2K_1$, admits linear prime labeling.

Example 2.4 $G = T_w(5)$.

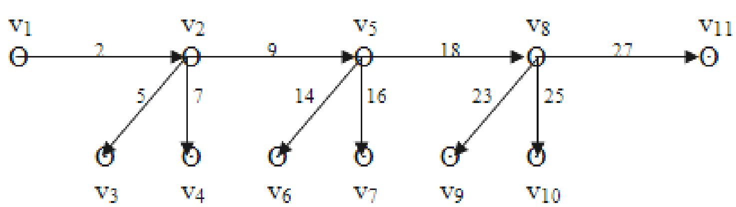

fig – 2.4

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Theorem 2.5 Direct graph of $P_n \Theta A(K_1)$ (n > 3) admits linear prime labeling, if n is even and pendant edges start from the first vertex.

Proof: Let $G = P_n \Theta A(K_1)$ and let $v_1, v_2, ---, v_{\frac{3n}{2}}$ are the vertices of G.

Here $|V(G)| = \frac{3n}{2}$ and $|E(G)| = \frac{3n-2}{2}$.

Define a function $f : V \rightarrow \{0,1,2,---, \frac{3n-2}{2}.\}$ by

$$f(v_i) = i-1 \text{ , } i = 1,2,---, \frac{3n}{2}$$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{lpl}^* is defined as follows

$f_{lpl}^*(v_1\, v_2) = 2.$

$f_{lpl}^*(v_2\, v_3) = 5.$

$f_{lpl}^*(v_{3i}\, v_{3i+1}) = 9i-1, \quad i = 1,2,---, \frac{n-2}{2}$

$f_{lpl}^*(v_{3i+1}\, v_{3i+2}) = 9i+2, \quad i = 1,2,---, \frac{n-2}{2}$

$f_{lpl}^*(v_{3i+1}\, v_{3i+3}) = 9i+4, \quad i = 1,2,---, \frac{n-2}{2}$

Clearly f_{lpl}^* is an injection.

In degree of each vertex of G is less than 2.

Hence $P_n \Theta A(K_1)$, admits linear prime labeling.

Example 2.5 $G = P_6 \Theta A(K_1)$.

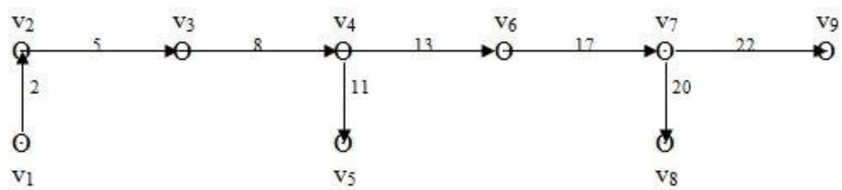

fig – 2.5

Theorem 2.6 Direct graph of $P_n \Theta A(K_1)$ (n > 3) admits linear prime labeling, if n is odd and pendant edges start from the first vertex.

Proof: Let $G = P_n \Theta A(K_1)$ and let $v_1, v_2, ---, v_{\frac{3n+1}{2}}$ are the vertices of G.

Here $|V(G)| = \frac{3n+1}{2}$ and $|E(G)| = \frac{3n-1}{2}$.

Define a function $f : V \rightarrow \{0,1,2,---, \frac{3n-1}{2}\}$ by

$$f(v_i) = i-1 \text{ , } i = 1,2,---, \frac{3n+1}{2}$$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{lpl}^* is defined as follows

$f_{lpl}^*(v_1\, v_2) = 2.$

$f_{lpl}^*(v_2\, v_3) = 5.$

$f_{lpl}^*(v_{3i}\, v_{3i+1}) = 9i-1, \quad i = 1,2,---, \frac{n-1}{2}$

$f_{lpl}^*(v_{3i+1}\, v_{3i+2}) = 9i+2, \quad i = 1,2,---, \frac{n-1}{2}$

$f_{lpl}^*(v_{3i+1}\, v_{3i+3}) = 9i+4, \quad i = 1,2,---, \frac{n-3}{2}$

Clearly f_{lpl}^* is an injection.

In degree of each vertex of G is less than 2.

Hence $P_n \Theta A(K_1)$, admits linear prime labeling.

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Example 2.6 G = $P_5 \odot A(K_1)$.

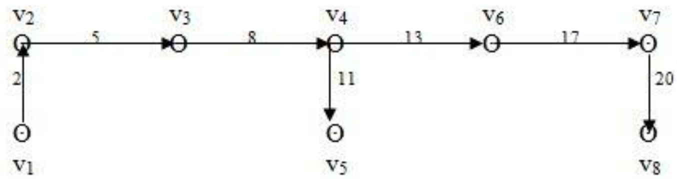

fig – 2.6

Theorem 2.7 Direct graph of $P_n \odot A(K_1)$ (n > 3) admits linear prime labeling, if n is odd and pendant edges start from the second vertex.

Proof: Let G = $P_n \odot A(K_1)$ and let $v_1, v_2, ---, v_{\frac{3n-1}{2}}$ are the vertices of G.

Here $|V(G)| = \frac{3n-1}{2}$ and $|E(G)| = \frac{3n-3}{2}$.

Define a function f : V → {0,1,2,---, $\frac{3n-3}{2}$.} by

$$f(v_i) = i-1 ,\ i = 1,2,---, \frac{3n-1}{2}$$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{lpl} is defined as follows

$f^*_{lpl}(v_1\ v_2)$	= 2.	
$f^*_{lpl}(v_{3i-1}\ v_{3i})$	= 9i-4,	i = 1,2,---, $\frac{n-1}{2}$
$f^*_{lpl}(v_{3i-1}\ v_{3i+1})$	= 9i-2,	i = 1,2,---, $\frac{n-1}{2}$
$f^*_{lpl}(v_{3i+1}\ v_{3i+2})$	= 9i+2,	i = 1,2,---, $\frac{n-3}{2}$

Clearly f^*_{lpl} is an injection.

In degree of each vertex of G is less than 2.

Hence $P_n \odot A(K_1)$, admits linear prime labeling.

Example 2.7 G = $P_5 \odot A(K_1)$.

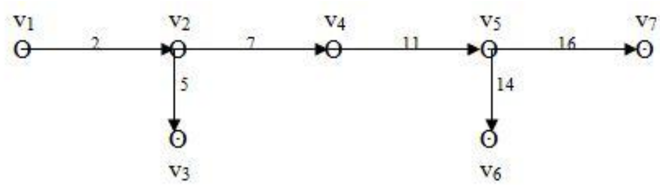

fig – 2.7

Theorem 2.8 Direct coconut tree graph CT(m,n) (m,n > 2) admits linear prime labeling.

Proof: Let G = CT(m,n) and let $v_1, v_2, ---, v_{m+n}$ are the vertices of G.

Here |V(G)| = m+n and |E(G)| = m+n-1.

Define a function f : V → {0,1,2,---,m+n-1} by

$$f(v_i) = i-1 ,\ i = 1,2,---,m+n.$$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{lpl} is defined as follows

$f^*_{lpl}(v_i\ v_{i+1})$	= 3i-1,	i = 1,2,---,m-1.
$f^*_{lpl}(v_m\ v_{m+i})$	= 3m+2i-3,	i = 1,2,---,n.

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Clearly f_{lpl}^* is an injection.
In degree of each vertex of G is less than 2.
Hence CT(m,n), admits linear prime labeling.

Example 2.8 G = CT(4,3)

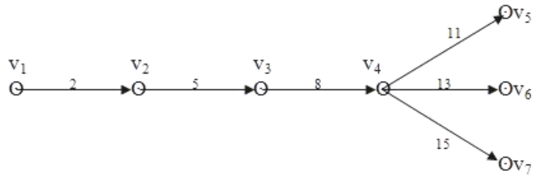

fig – 2.8

Theorem 2.9 Direct double coconut tree graph DCT(m, n, k) (m, n, k > 2) admits linear prime labeling.

Proof: Let G = DCT(m, n, k) and let $v_1,v_2,---,v_{m+n+k}$ are the vertices of G.
Here |V(G)| = m+n+k and |E(G)| = m+n+k-1.
Define a function f : V → {0,1,2,---,m+n+k-1} by
$f(v_i)$ = i-1 , i = 1,2,---,m+n+k.
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f_{lpl}^* is defined as follows

$f_{lpl}^*(v_i\ v_{m+1})$ = 2m+i-1, i = 1,2,---,m.
$f_{lpl}^*(v_{m+i}\ v_{m+i+1})$ = 3m+3i-1, i = 1,2,---,n-1.
$f_{lpl}^*(v_{m+n}\ v_{m+n+i})$ = 3m+3n+2i-3, i = 1,2,---,k.

Clearly f_{lpl}^* is an injection.
gcin of (v_{m+1}) = gcd of $\{f_{lpl}^*(v_1\ v_{m+1}), f_{lpl}^*(v_2\ v_{m+1})\}$
= gcd of {2m, 2m+1}
= 1.
So, ***gcin*** of each vertex of in degree greater than one is 1.
Hence DCT(m, n, k), admits linear prime labeling.

Example 2.9 G = DCT(3,4,5)

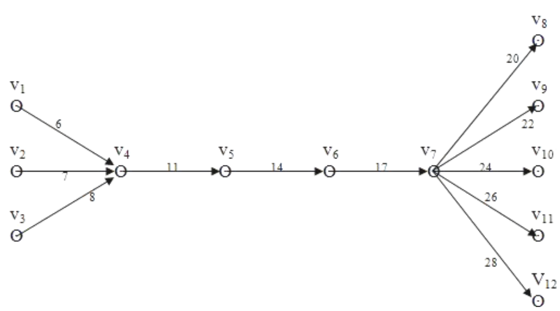

fig – 2.9

Theorem 2.10 Direct banana tree graph B(n,k) (n, k > 2) admits linear prime labeling.

Proof: Let G = B(n,k) and let $v_1,v_2,---,v_{nk+n+1}$ are the vertices of G.
Here |V(G)| = nk+n+1 and |E(G)| = nk+n.
Define a function f : V → {0,1,2,---,nk+n} by

f(a) = 0

$f(v_i) = i$, i = 1,2,---,nk+n.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{lpl} is defined as follows

$f^*_{lpl}(a\ v_i)$	= 2i,	i = 1,2,---,n.
$f^*_{lpl}(v_j\ v_{n+(j-1)k+i})$	= 2n+2(j-1)k+2i+j,	j = 1,2,---,n. i = 1,2,---,k.

Clearly f^*_{lpl} is an injection.

In degree of each vertex of G is less than 2.

Hence B(n,k) , admits linear prime labeling.

Example 2.10 G = B(4, 3)

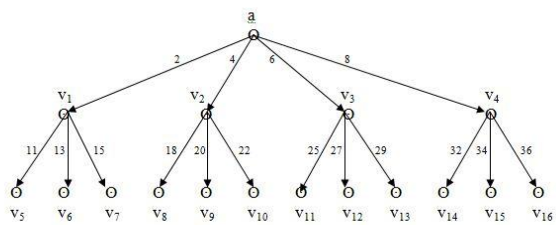

fig – 2.10

III. CONCLUSION

Labeling of direct graphs plays an important role in the study of network related problems. Here we proved that direct comb graph, direct centipede graph, direct twig graph, direct hurdle graph, direct graph of the graph obtained by joining pendant edges alternately to the vertices of a path, direct coconut tree graph, direct double coconut tree graph and direct banana tree graphs admit linear prime labeling. Further research can be carried out in labeling the direct fire cracker graph and more tree graphs.

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